Parton Saturation and Color Glass Condensate

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The Main Idea of Saturation/CGC Physics

- Saturation physics is based on the existence of a large internal momentum scale Q_S which grows with both energy and nuclear atomic number A

$$\boxed{Q_S^2 \sim \Lambda_{QCD}^2 \; A^{1/3} \left(\frac{E}{\Lambda_{QCD}} \right)^{\Delta}}$$

such that at high energies E or for large nuclei with

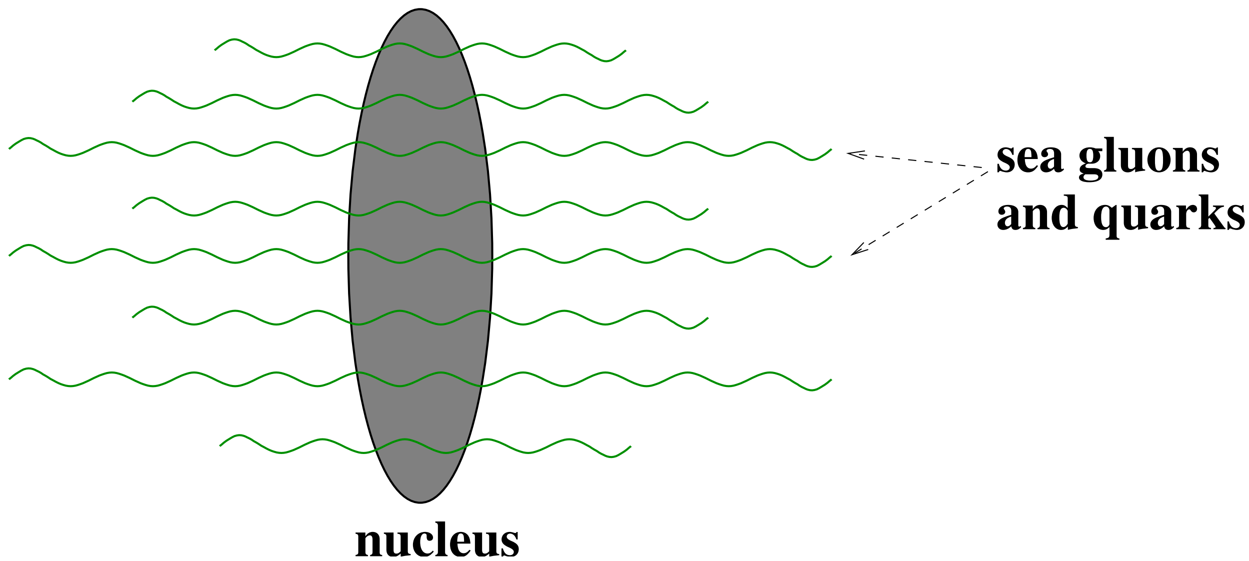
A >> 1: $Q_S \gg \Lambda_{QCD}$ and

$$\alpha_S(Q_S) \ll 1.$$

Don't need to look for rare processes to quantify things: can calculate total cross sections from first principles!

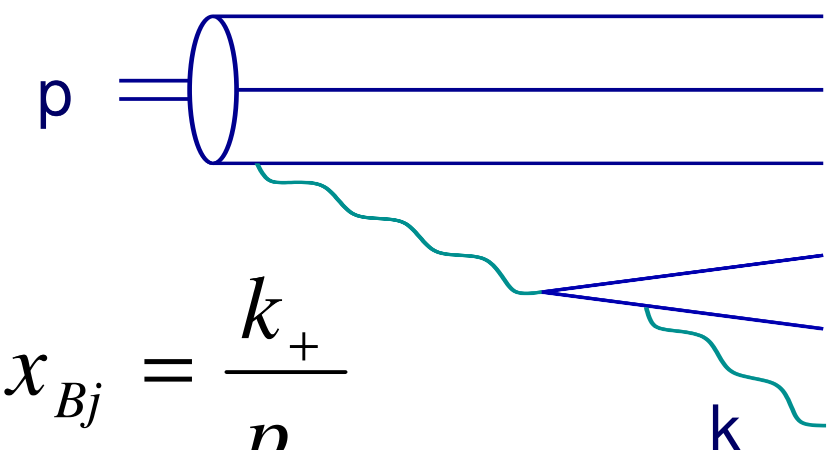
Classical Fields

Nuclear/Hadronic Wave Function

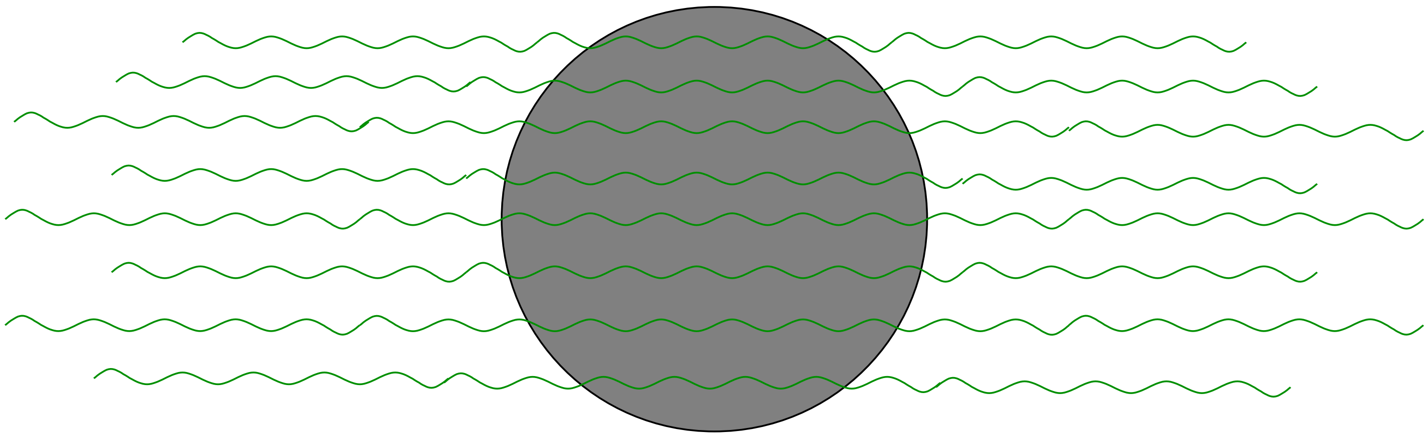


Imagine an UR nucleus or hadron with valence quarks and sea gluons and quarks. Define Bjorken (Feynman) x as

$$x_{Bj} = \frac{k_+}{p_+}$$



Rest frame of the hadron/nucleus



nucleus in the rest frame

Longitudinal coherence length (wavelength) of a gluon is

$$l_{coh} \sim \frac{1}{k_+} \sim \frac{1}{x_{Bj}\, p_+} \sim \frac{1}{x_{Bj}\, m_N}$$

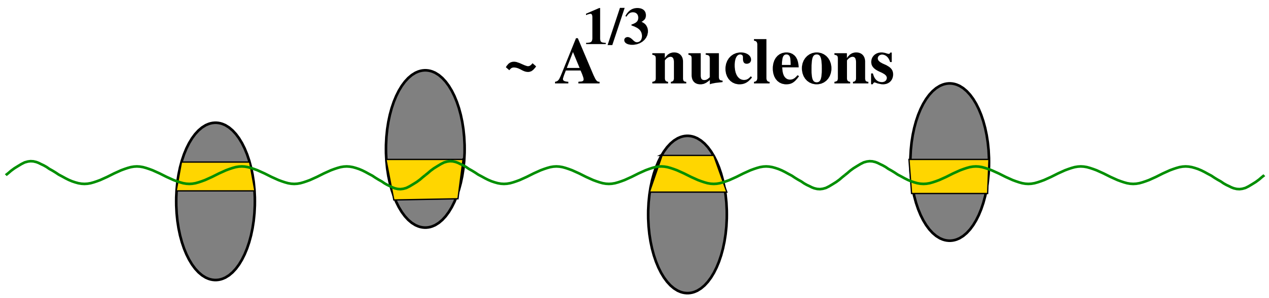
such that for small enough x_{Bj} we get

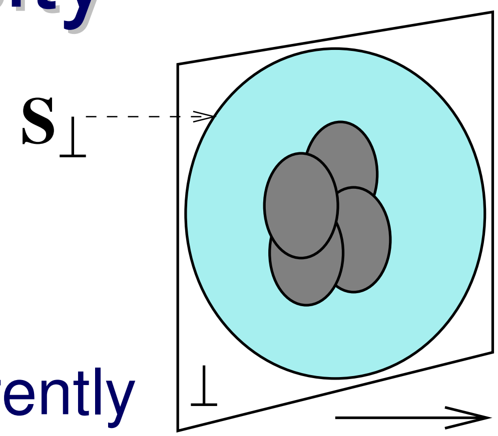
$$l_{coh} \approx \frac{1}{2\, m_N\, x_{Bj}} >> R$$

with R the nuclear radius.

(e.g. for $x=10^{-3}$ get l_{coh}=100 fm)

Color Charge Density





Small-x gluon "sees" the whole nucleus coherently in the longitudinal direction! It "sees" many color charges which form a net effective color charge Q = g (# charges)$^{1/2}$, such that $Q^2 = g^2$ #charges (random walk). Define color charge density

$$\mu^2 = \frac{Q^2}{S_\perp} = \frac{g^2 \, \# \, \text{charges}}{S_\perp} \sim g^2 \frac{A}{S_\perp} \sim A^{1/3}$$

McLerran
Venugopalan
'93-'94

such that for a large nucleus (A>>1)

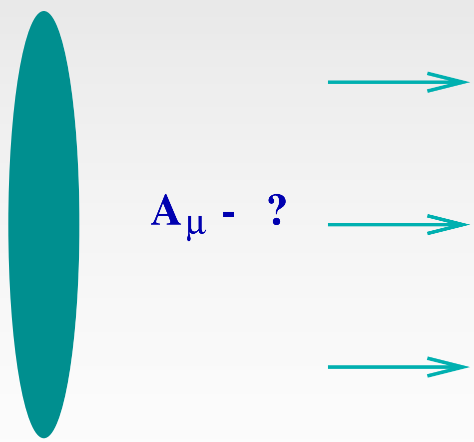
$$\boxed{\mu^2 \sim \Lambda_{QCD}^2 A^{1/3} >> \Lambda_{QCD}^2 \quad \Rightarrow \quad \alpha_S(\mu^2) << 1}$$

Nuclear small-x wave function is perturbative!!!

McLerran-Venugopalan Model

- The density of partons in the nucleus (number of partons per unit transverse area) is given by the scale $\mu^2 \sim A/\pi R^2$.
- This scale is large, $\mu >> \Lambda_{QCD}$, so that the strong coupling constant is small, $\alpha_S\,(\mu) << 1$.
- Leading gluon field is classical! To find the classical gluon field A_μ of the nucleus one has to solve the Yang-Mills equations, with the nucleus as a source of color charge:

$$D_\nu F^{\mu\nu} = J^\mu$$

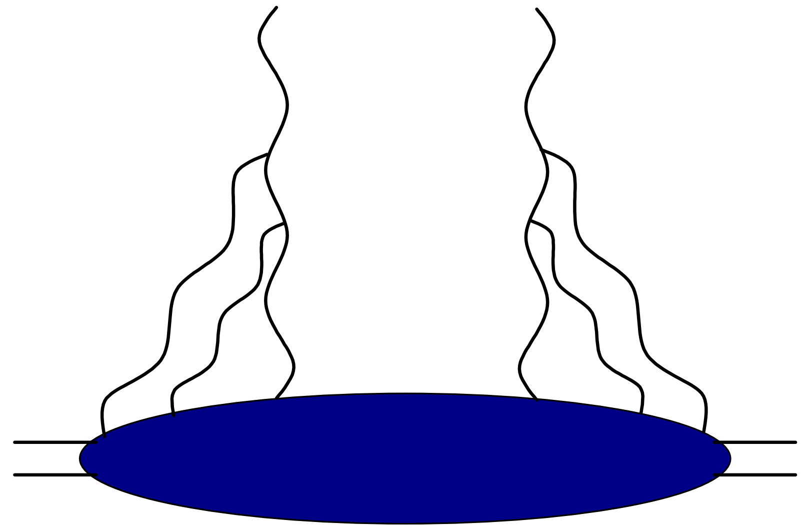


nucleus is Lorentz contacted into a pancake

Yu. K. '96

J. Jalilian-Marian et al, '96

Classical Gluon Field of a Nucleus



Using the obtained classical gluon field one can construct corresponding gluon distribution function $\phi_A(x,k^2) \sim \left\langle \underline{A}(-k) \cdot \underline{A}(k) \right\rangle$

getting

$$\phi_A(x,k^2) = \int d^2x \, e^{i\underline{k}\cdot\underline{x}} \frac{C_F}{\alpha_S \pi x_\perp^2} \left(1 - \exp\left[-\frac{x_\perp^2 Q_S^2}{4} \ln\left(\frac{1}{x_\perp \Lambda} \right) \right] \right)$$

J. Jalilian-Marian et al, ’97; Yu. K. and A. Mueller, ‘98

⇨ $Q_S = \mu$ is the saturation scale $\quad Q_S^2 \sim A^{1/3}$

⇨ Note that $\phi \sim \langle A_\mu A_\mu \rangle \sim 1/\alpha$ such that $A_\mu \sim 1/g$, which is what one would expect for a classical field.

$$\phi_A(x,k^2) = \int d^2x\, e^{i\underline{k}\cdot\underline{x}}\, \frac{C_F}{\alpha_S\,\pi\,x_\perp^2}\left(1-\exp\left[-\frac{x_\perp^2\,Q_S^2}{4}\ln\left(\frac{1}{x_\perp\,\Lambda}\right)\right]\right)$$

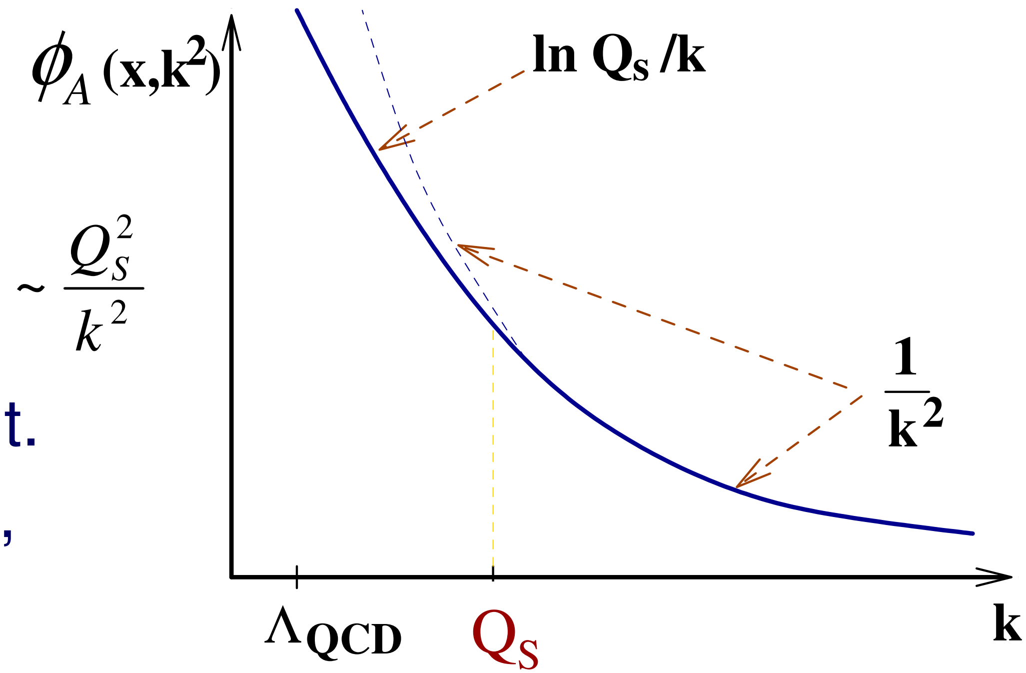
⇨ In the UV limit of k→∞, x_T is small and one obtains

$$\phi_A(x,k^2) \sim \int d^2x\, e^{i\underline{k}\cdot\underline{x}}\, Q_S^2 \ln\frac{1}{x_\perp\,\Lambda} \sim \frac{Q_S^2}{k^2}$$

which is the usual LO result.

⇨ In the IR limit of small k_T, x_T is large and we get

$$\phi_A(x,k^2) \sim \int_{1/Q_S} d^2x\, e^{i\underline{k}\cdot\underline{x}}\, \frac{1}{x_\perp^2} \sim \ln\frac{Q_S}{k}$$

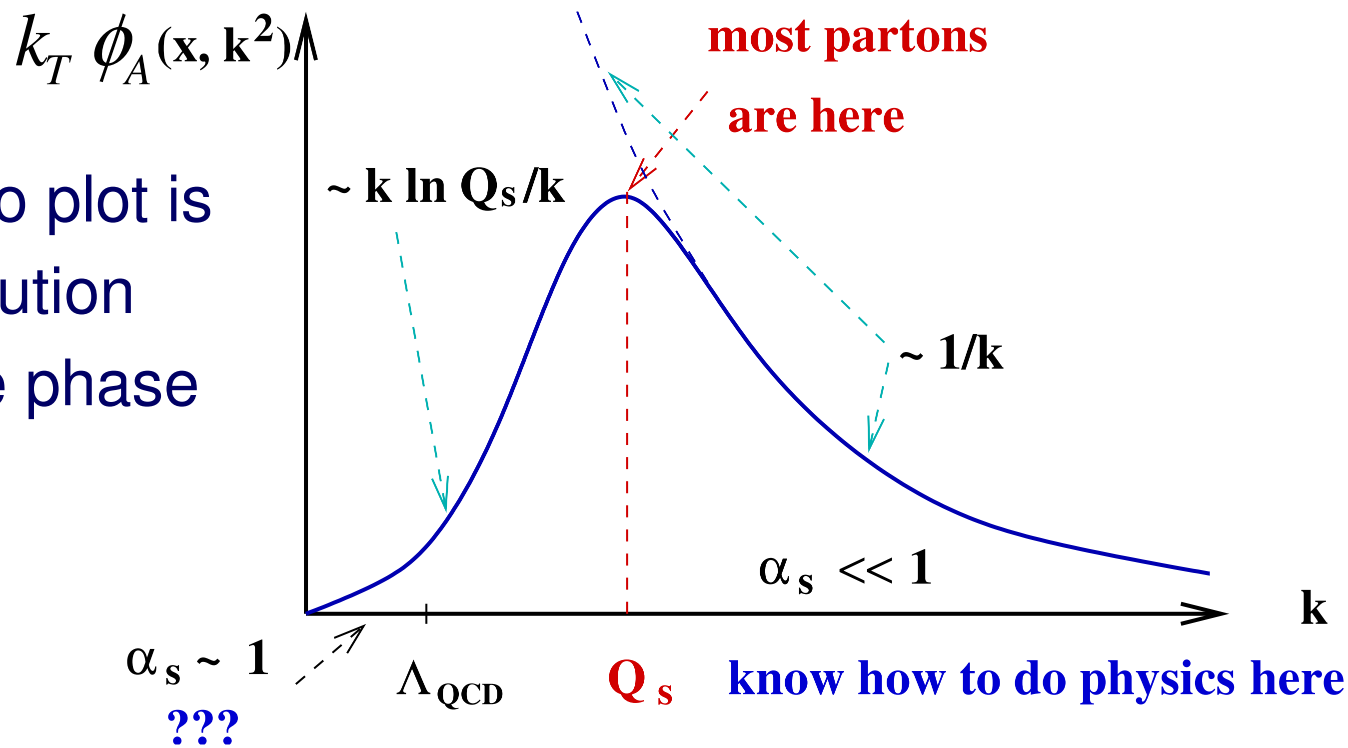


SATURATION !

Divergence is regularized.

Classical Gluon Distribution

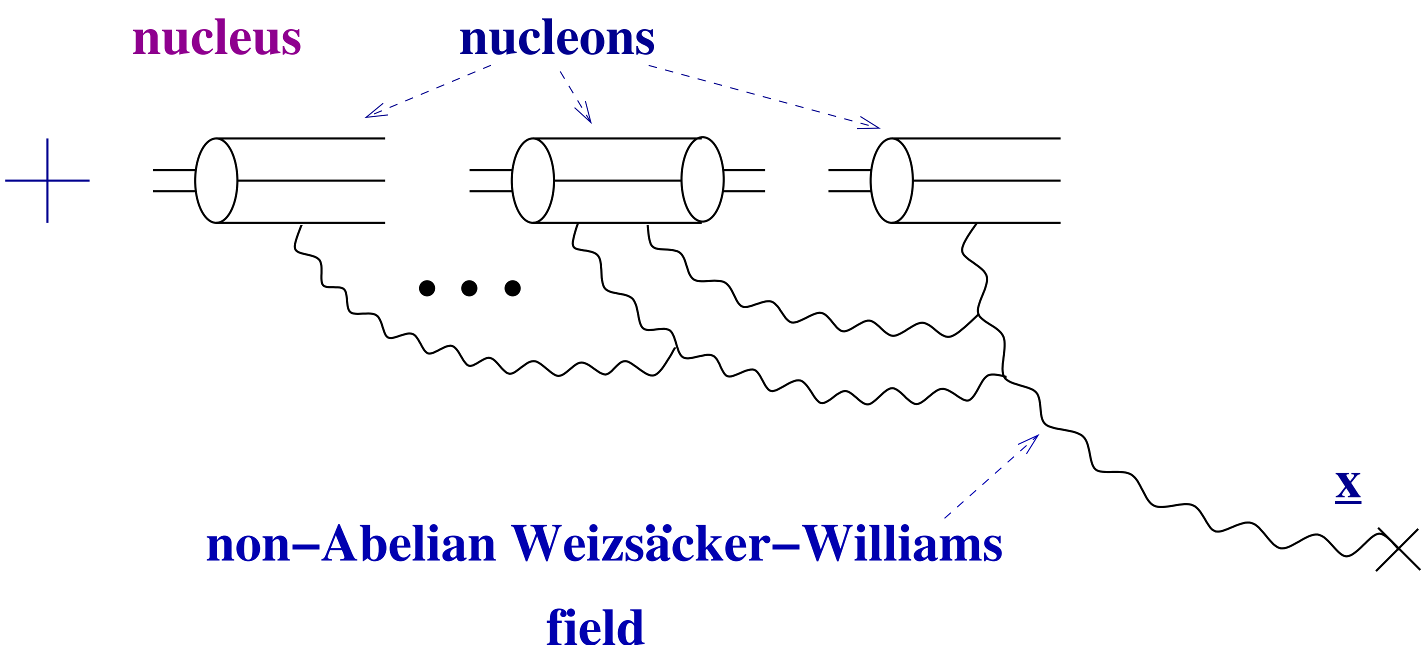
A better object to plot is the gluon distribution multiplied by the phase space k_T:



⇨ Most gluons in the nuclear wave function have transverse momentum of the order of $k_T \sim Q_S$ and $Q_S^2 \sim A^{1/3}$

⇨ We have a small coupling description of the whole wave function in the classical approximation.

Classical Field of a Nucleus



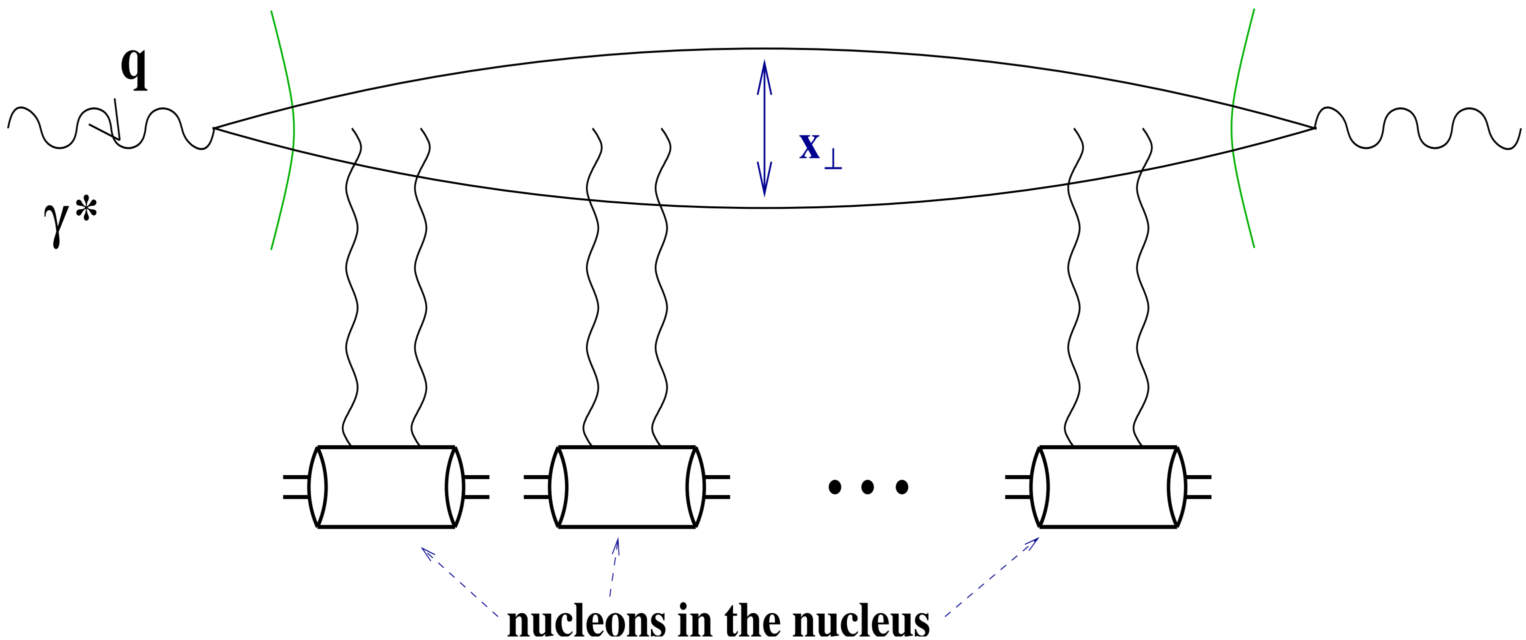
Here's one of the diagrams showing the non-Abelian Weizsacker-Williams field of a large nucleus.

The resummation parameter is $\alpha_S^2 \, A^{1/3}$, corresponding to two gluons per nucleon approximation.

Quantum Evolution

DIS in the Classical Approximation

The DIS process in the rest frame of the target is shown below. It factorizes into



$$\sigma_{tot}^{\gamma^* A}(x_{Bj}, Q^2) = \Phi^{\gamma^* \to q\bar{q}} \otimes N(x_\perp, Y = \ln(1/x_{Bj}))$$

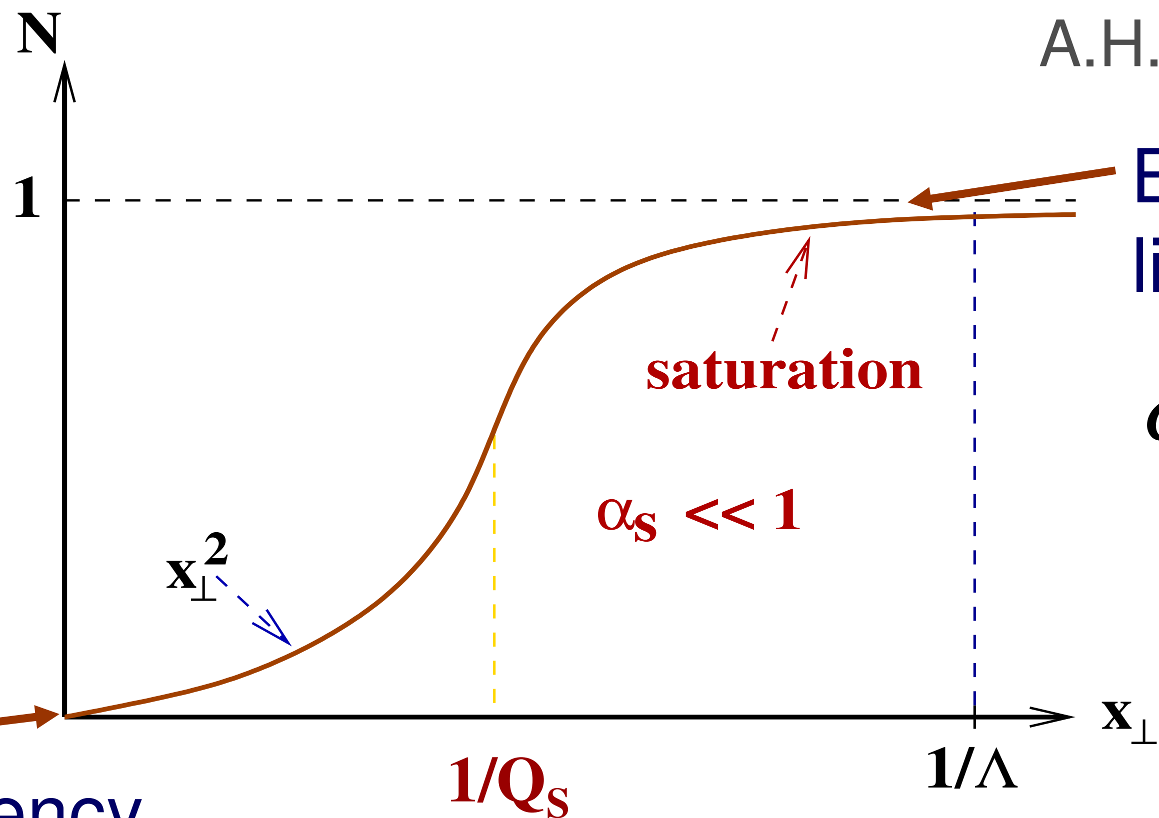
with rapidity Y=ln(1/x)

DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[-\frac{x_\perp^2 Q_S^2}{4} \ln \frac{1}{x_\perp \Lambda} \right]$$

A.H. Mueller, '90



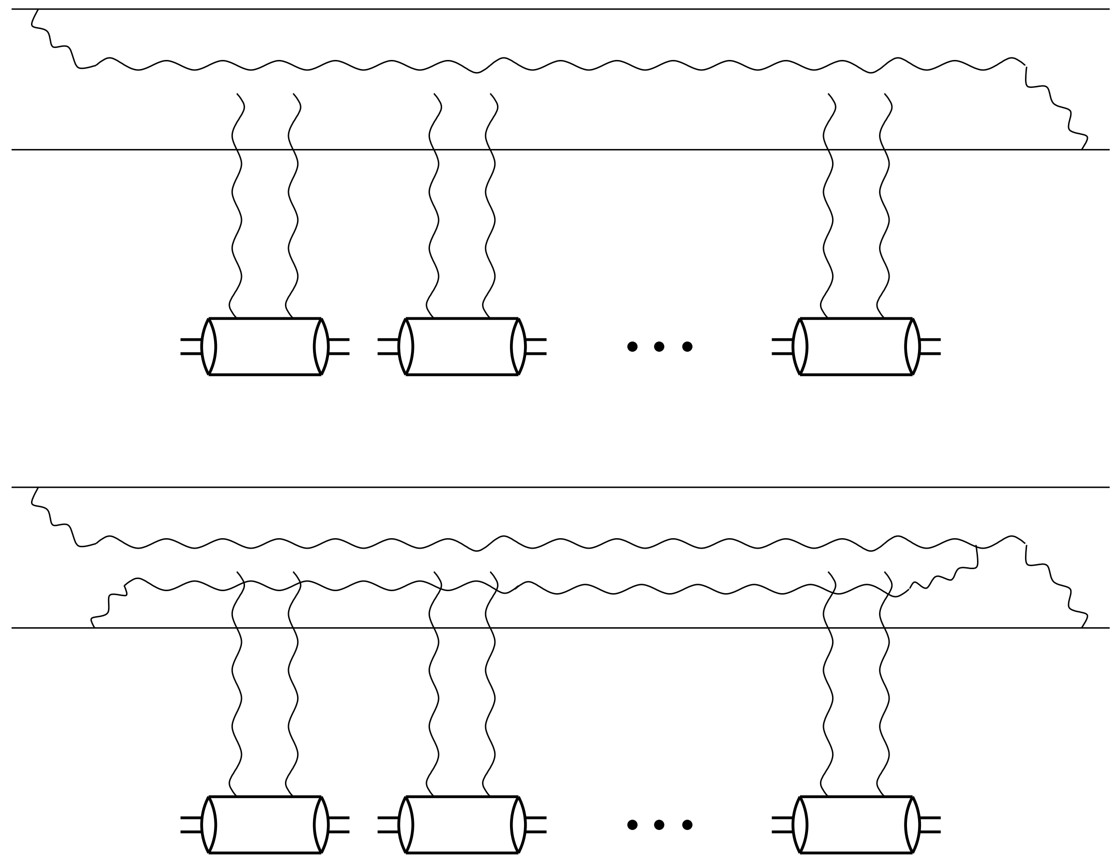
Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

Quantum Evolution

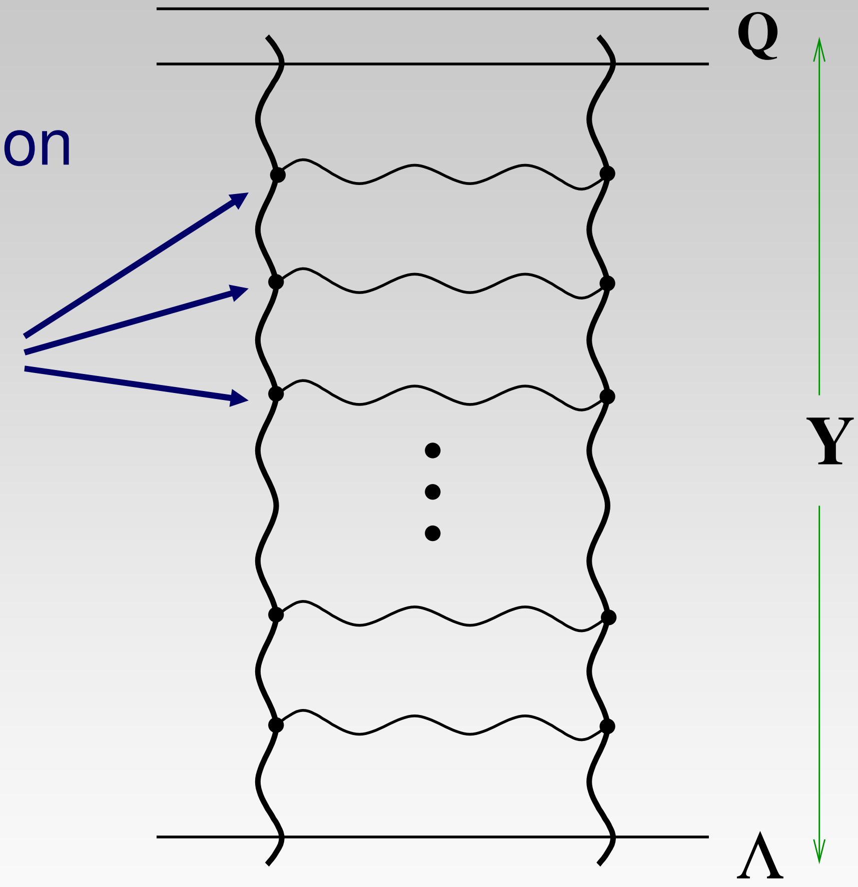
As energy increases the higher Fock states including gluons on top of the quark-antiquark pair become important. They generate a cascade of gluons.



These extra gluons bring in powers of $\alpha_s \ln s$, such that when $\alpha_s << 1$ and $\ln s >>1$ this parameter is $\alpha_s \ln s \sim 1$.

BFKL Equation

In the conventional Feynman diagram picture the BFKL equation can be represented by a ladder graph shown here. Each rung of the ladder brings in a power of $\alpha \ln s$.



The resulting dipole amplitude grows as a power of energy

$$N \sim s^{\Delta}$$

violating Froissart unitarity bound

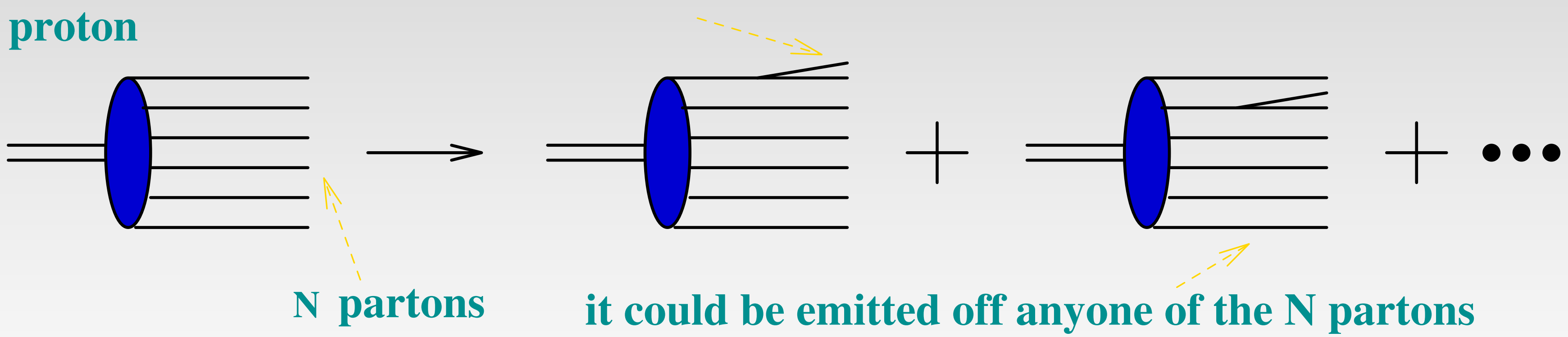
$$\sigma_{tot} \leq const \ln^2 s$$

How can we fix the problem? Let's first resum the cascade of gluons shown before.

BFKL Equation

Balitsky, Fadin, Kuraev, Lipatov '78

The powers of the parameter α ln s without multiple rescatterings are resummed by the BFKL equation. Start with N particles in the proton's wave function. As we increase the energy a new particle can be emitted by either one of the N particles. The number of newly emitted particles is proportional to N.



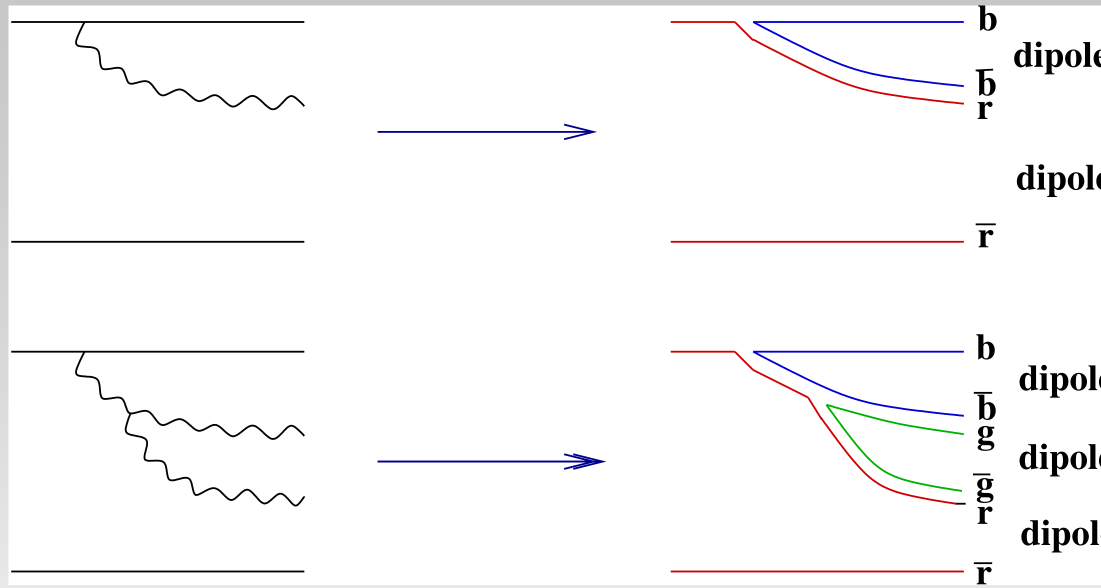
The BFKL equation for the number of partons N reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x, Q^2) = \alpha_S K_{BFKL} \otimes N(x, Q^2)$$

Resumming Gluonic Cascade

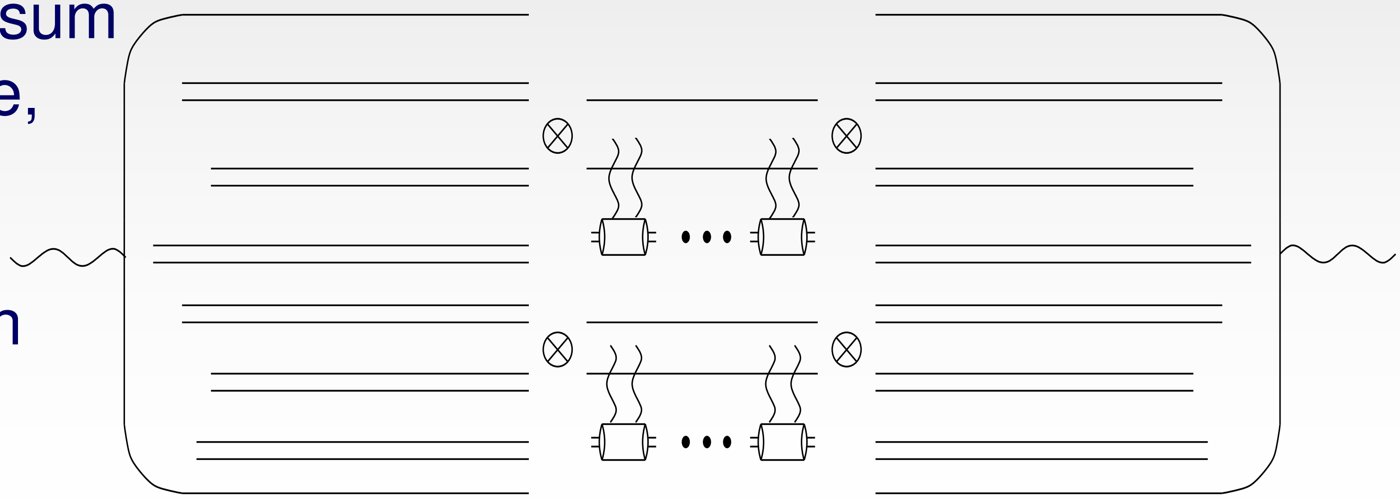
In the large-N_C limit of QCD the gluon corrections become color dipoles. Gluon cascade becomes a dipole cascade.

A. H. Mueller, '93-'94

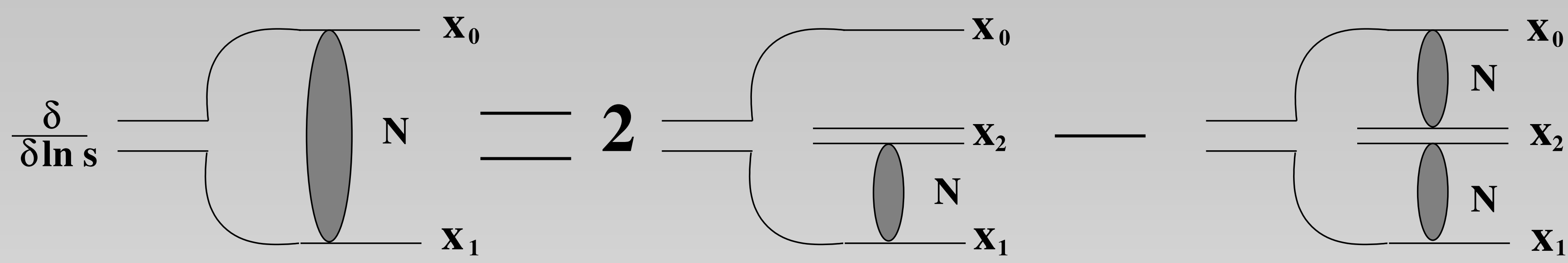


We need to resum dipole cascade, with each final state dipole interacting with the target.

Yu. K. '99



Nonlinear Evolution Equation



Defining rapidity Y=ln s we can resume the dipole cascade

$$\frac{\partial N(x_{01},Y)}{\partial Y} = \frac{\alpha_S N_C}{\pi^2}\int d^2x_2 \left[\frac{x_{01}^2}{x_{02}^2\, x_{12}^2} - 2\pi\,\delta^2(\underline{x}_{01}-\underline{x}_{02})\ln\left(\frac{x_{01}}{\rho}\right)\right] N(x_{02},Y)$$

$$- \frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2} N(x_{02},Y)\, N(x_{12},Y)$$

Yu. K., ’99, large N_C QCD

I. Balitsky, ’96, HE effective lagrangian

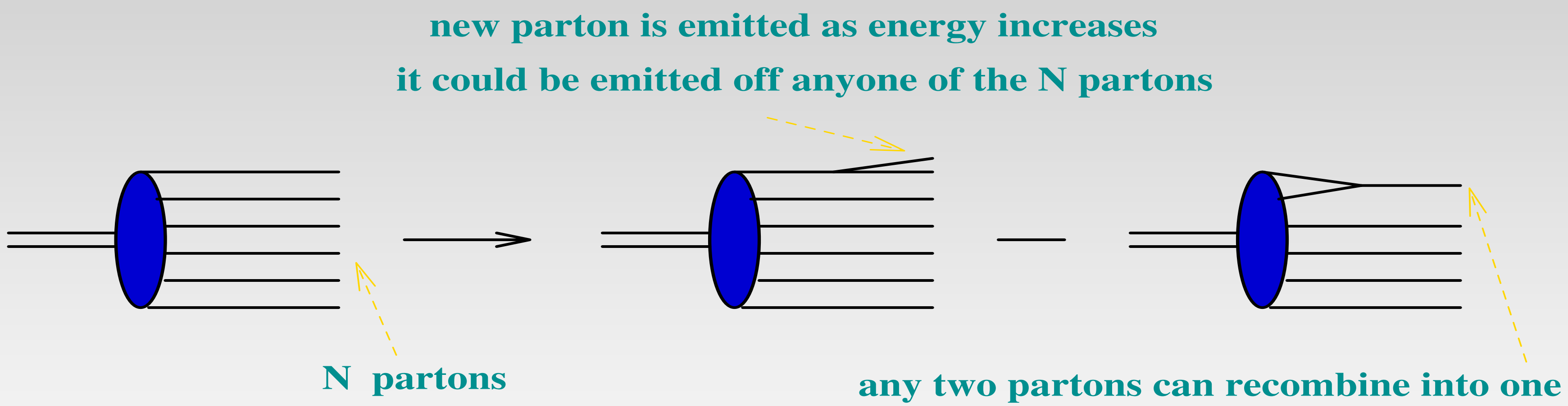
$$N(x_\perp, Y=0) = 1 - \exp\left[-\frac{x_\perp^2 Q_S^2}{4}\ln\frac{1}{x_\perp \Lambda}\right]$$

initial condition

⇨ Linear part is BFKL, quadratic term brings in damping

Nonlinear Equation

At very high energy parton recombination becomes important. Partons not only split into more partons, but also recombine. Recombination reduces the number of partons in the wave function.



$$\frac{\partial N(x,k^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,k^2) - \alpha_s [N(x,k^2)]^2$$

Number of parton pairs ~ N^2

Yu. K. '99 (large N_C QCD)
I. Balitsky '96 (effective lagrangian)

Going Beyond Large N_C: JIMWLK

To do calculations beyond the large-N_C limit on has to use a functional integro-differential equation written by Iancu, Jalilian-Marian, Kovner, Leonidov, McLerran and Weigert (JIMWLK):

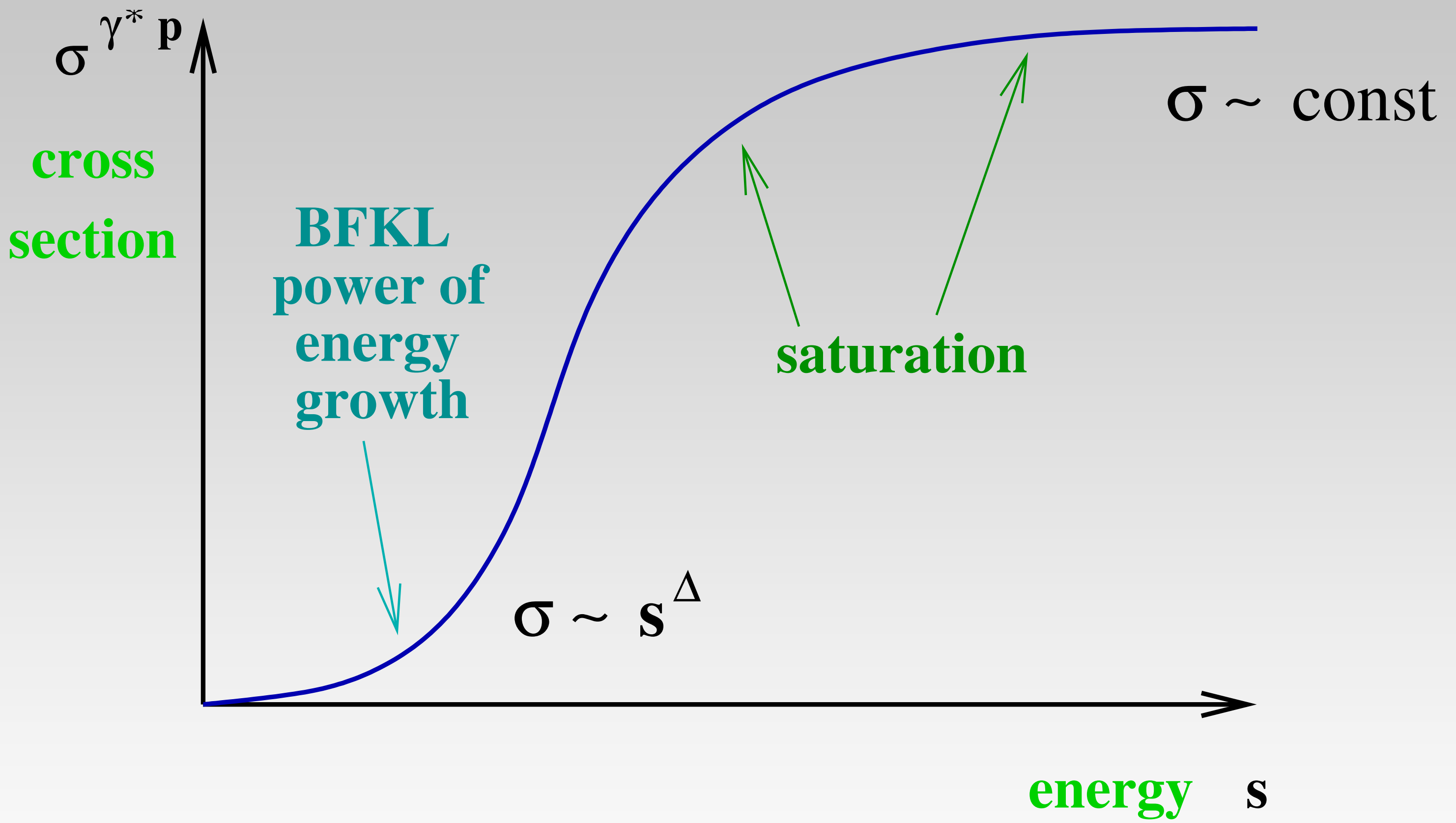
$$\boxed{\frac{\partial Z}{\partial Y} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho(u)\,\delta\rho(v)} [Z\,\chi(u,v)] - \frac{\delta}{\delta\rho(u)} [Z\,\sigma(u)] \right\}}$$

where the functional Z[ρ] can then be used for obtaining wave function-averaged observables (like Wilson loops for DIS):

$$< O >= \frac{\int D\rho\, Z[\rho] O[\rho]}{\int D\rho\, Z[\rho]}$$

A lot of progress on solving JIMWLK on the lattice has recently been achieved by K. Rummukainen and H. Weigert

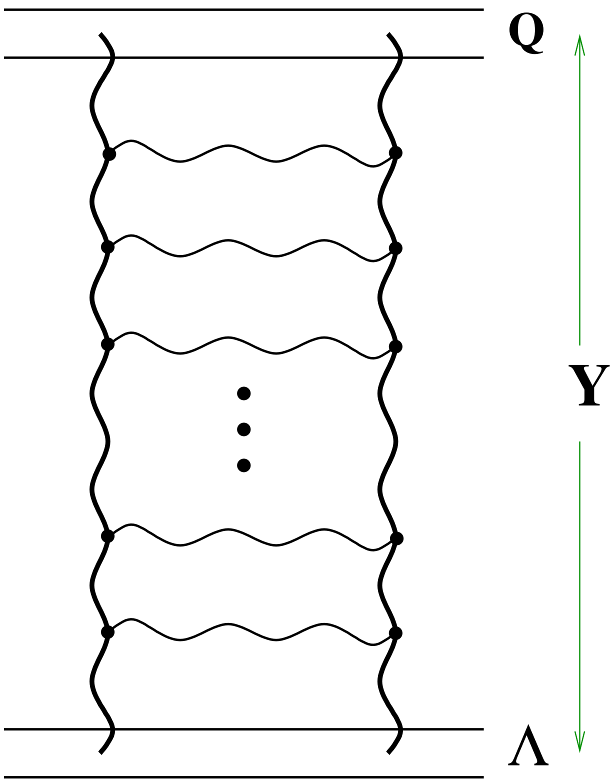
Nonlinear Equation: Saturation



Gluon recombination tries to reduce the number of gluons in the wave function. At very high energy recombination begins to compensate gluon splitting. Gluon density reaches a limit and does not grow anymore. So do total DIS cross sections. Unitarity is restored!

Energy Dependence of the Saturation Scale

Single BFKL ladder gives scattering amplitude of the order $N \sim \frac{\Lambda}{k} s^{\Delta}$



Nonlinear saturation effects become important when N ~ N² ⇨ N ~ 1. This happens at $\boxed{k_T = Q_S \sim \Lambda\, s^{\Delta}}$

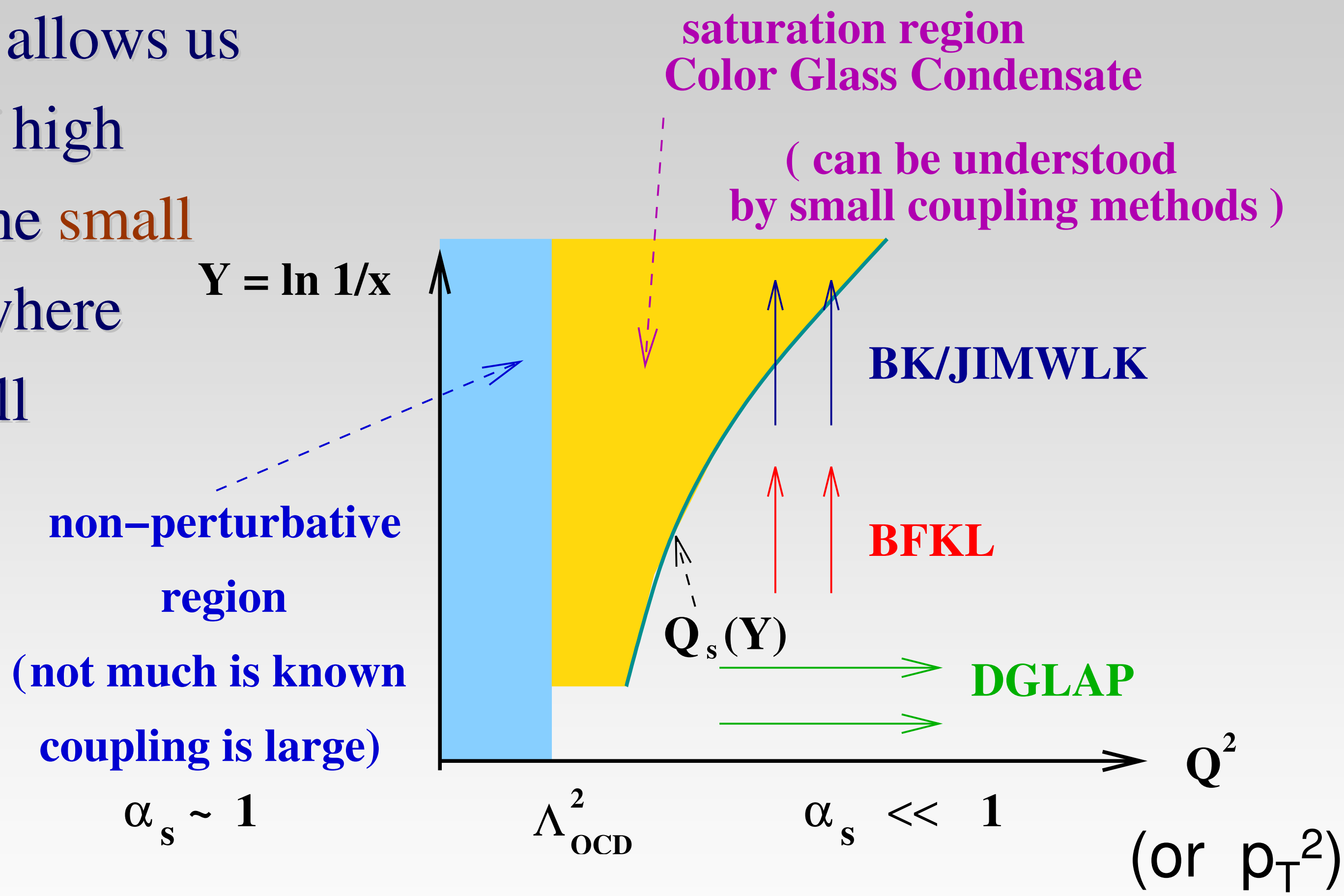
Saturation scale grows with energy!

Typical partons in the wave function have $k_T \sim Q_S$, so that their characteristic size is of the order $r \sim 1/k_T \sim 1/Q_S$.

⇨ Typical parton size decreases with energy!

"Phase Diagram" of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!



Transition to saturation region is characterized by the <u>saturation scale</u>

What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_{01},Y)}{\partial Y} = \frac{\alpha_S N_C}{\pi^2} \int d^2x_2 \left[\frac{x_{01}^2}{x_{02}^2\, x_{12}^2} - 2\pi\, \delta^2(\underline{x}_{01} - \underline{x}_{02}) \ln\left(\frac{x_{01}}{\rho}\right) \right] N(x_{02},Y)$$

$$- \frac{\alpha_S N_C}{2\pi^2} \int d^2x_2 \frac{x_{01}^2}{x_{02}^2\, x_{12}^2} N(x_{02},Y)\, N(x_{12},Y)$$

$\alpha_S(???)$

In order to perform consistent calculations it is important to know the scale of the running coupling constant in the evolution equation.

There are three possible scales – the sizes of "parent" dipole and "daughter" dipoles x_{01}, x_{21}, x_{20}. Which one is it?

Coming up soon – E. Gardi, K. Rummukainen, J. Kuokkanen, H. Weigert, Yu. K.

Geometric Scaling

✓ Geometric scaling is the property of the solution of nonlinear evolution equation. The solution leads to, say, gluon distribution being a function of just one variable

$$\phi^A(k,y) = \phi^A\left(\frac{k}{Q_S(y)}\right), \qquad k < Q_S$$

inside the saturation region (Levin, Tuchin '99) and beyond

$$\phi^A(k,y) = \phi^A\left(\frac{k}{Q_S(y)}\right), \qquad k < k_{geom}$$

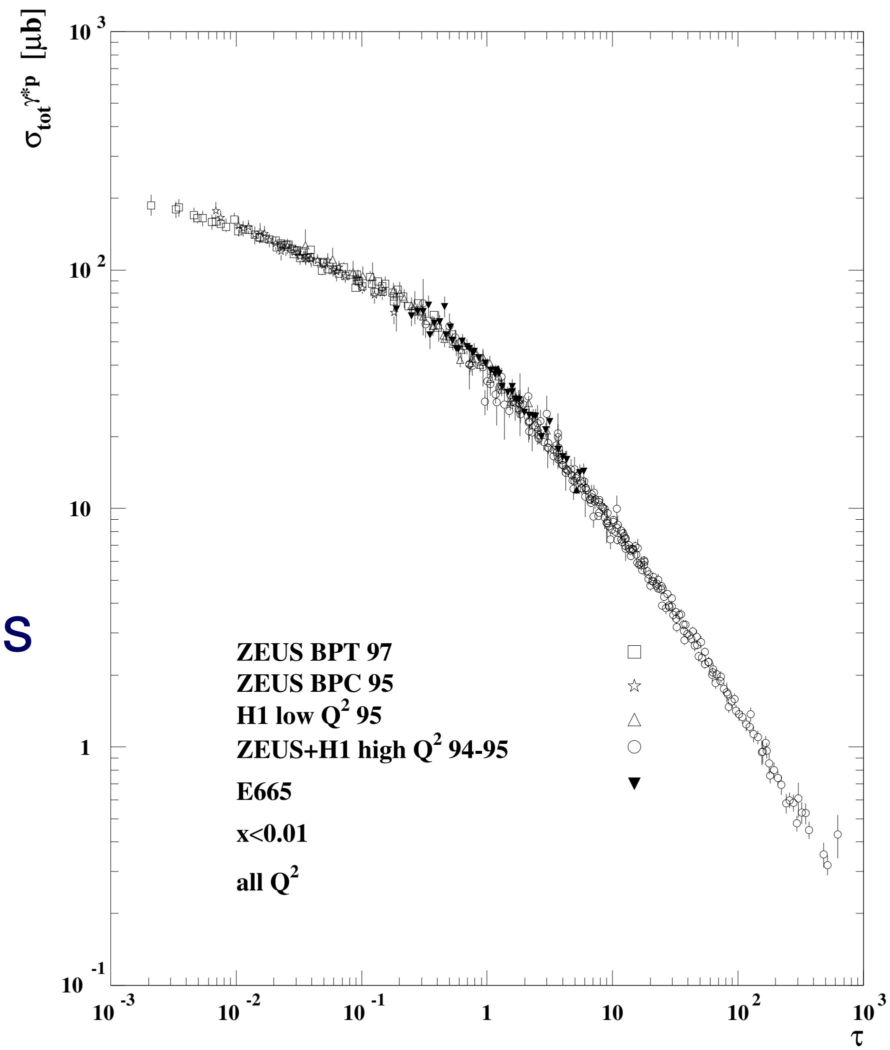
(Iancu, Itakura, McLerran '02).
The latter extension is called extended geometric scaling.

Geometric Scaling in DIS

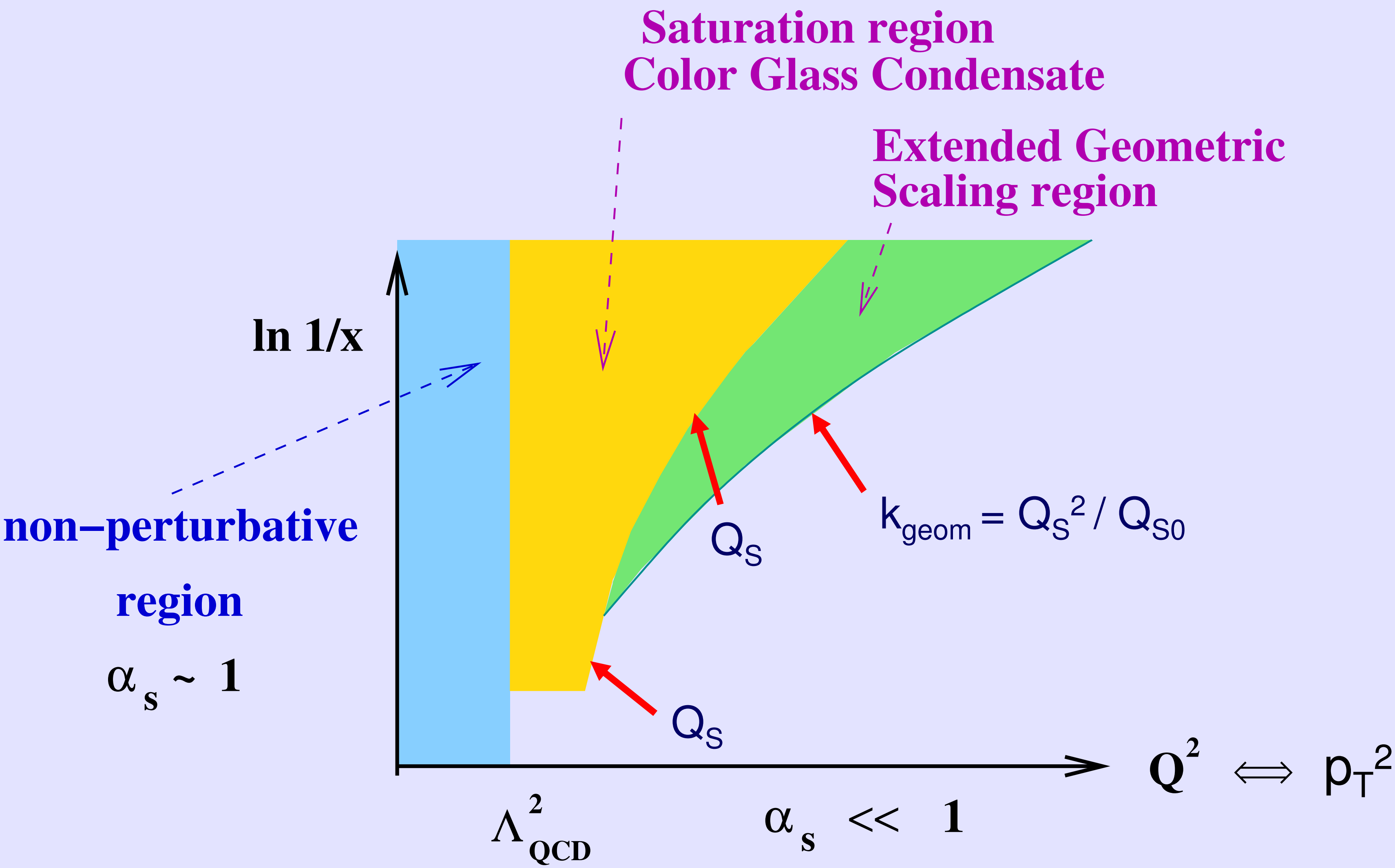
Geometric scaling has been observed in DIS data by Stasto, Golec-Biernat, Kwiecinski in '00.

Here they plot the total DIS cross section, which is a function of 2 variables - Q^2 and x, as a function of just one variable:

$$\tau = \frac{Q^2}{Q_S^2(x)}$$



“Phase Diagram” of High Energy QCD



Particle Production

How to Calculate Observables

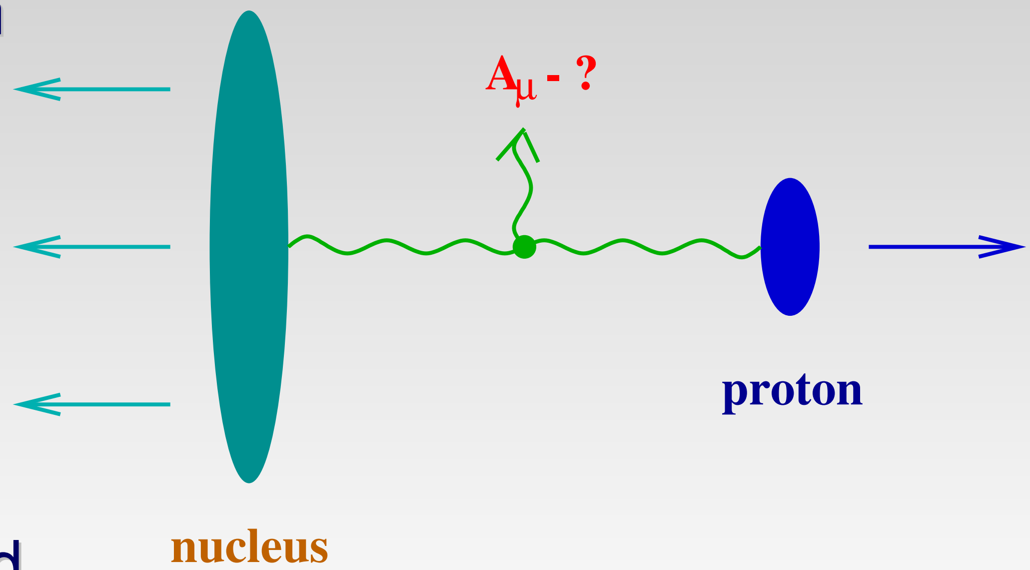
- Start by finding the classical field of the McLerran-Venugopalan model.
- Continue by including the quantum corrections of the nonlinear evolution equation.
- Works for structure functions of hadrons and nuclei, as well as for gluon production in various hadronic collisions. Let us consider pA collisions first.

Gluon Production in Proton-Nucleus Collisions (pA): Classical Field

To find the gluon production cross section in pA one has to solve the same classical Yang-Mills equations

$$D_\nu F^{\mu\nu} = J^\mu$$

for two sources – proton and nucleus.

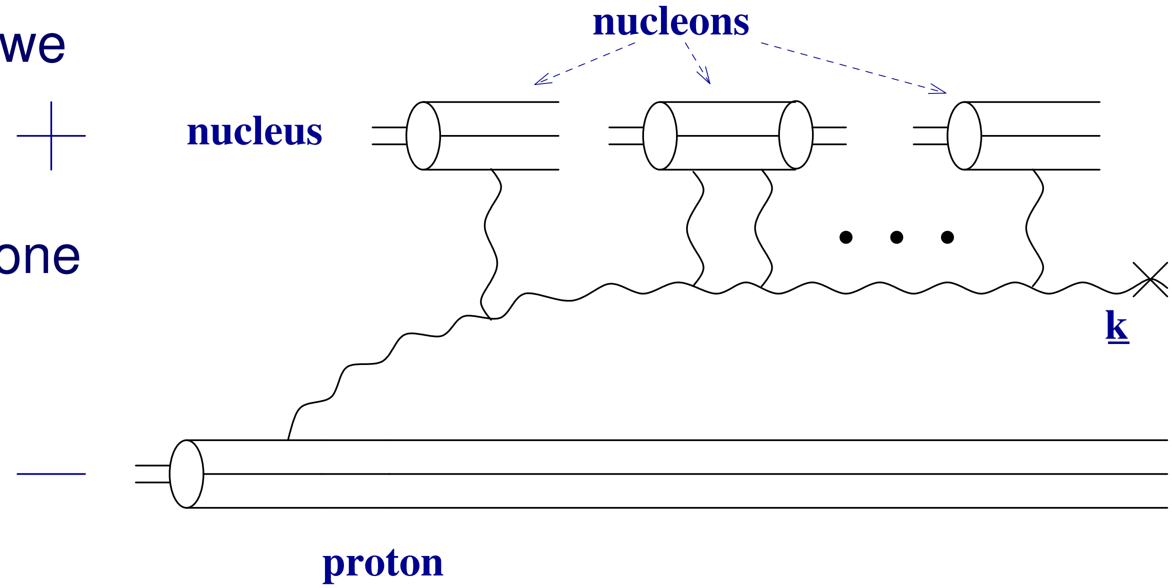


This classical field has been found by Yu. K., A.H. Mueller in '98

Gluon Production in pA: McLerran-Venugopalan model

Classical gluon production: we need to resum only the multiple rescatterings of the gluon on nucleons. Here's one of the graphs considered.

Yu. K., A.H. Mueller, hep-ph/9802440



Resulting inclusive gluon production cross section is given by

$$\frac{d\sigma}{d^2k\,dy} = \frac{1}{(2\pi)^2}\int d^2b\,d^2x\,d^2y\,e^{i\underline{k}\cdot(\underline{x}-\underline{y})}\underbrace{\frac{\alpha C_F}{\pi^2}\frac{\underline{x}\cdot\underline{y}}{x^2\,y^2}}_{\text{proton's wave function}}\left[N_G(x)+N_G(y)-N_G(\underline{x}-\underline{y})\right]$$

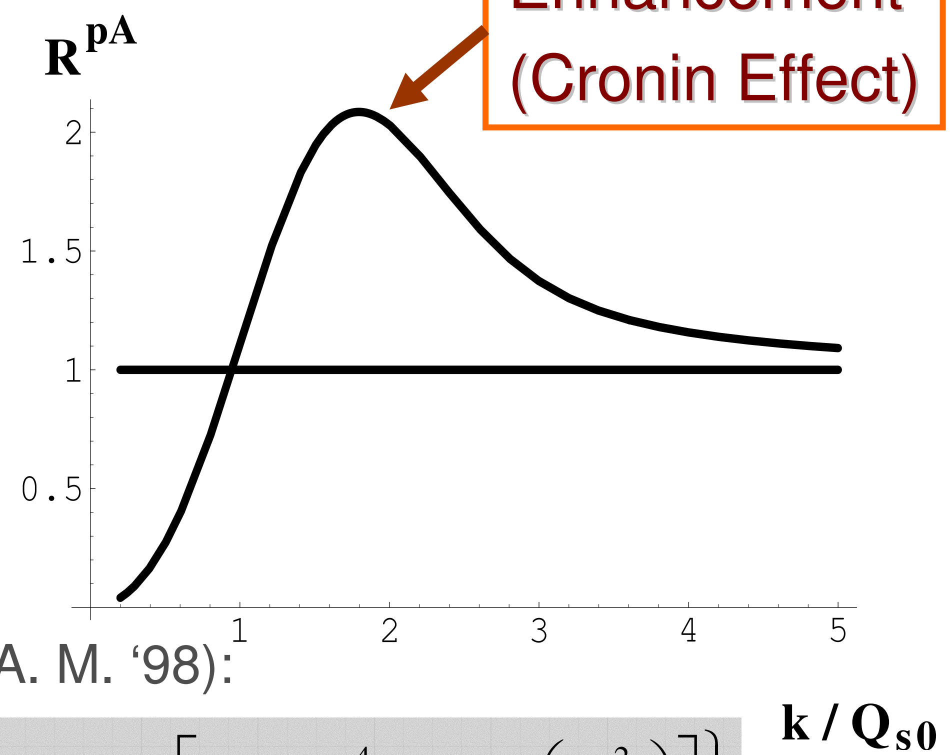
With the gluon-gluon dipole-nucleus forward scattering amplitude

$$N_G(x, Y=0) = 1 - e^{-x^2 Q_s^2/4}$$

McLerran-Venugopalan model: Cronin Effect

To understand how the gluon production in pA is different from independent superposition of *A* proton-proton (pp) collisions one constructs the quantity

$$R^{pA} = \frac{\dfrac{d\sigma^{pA}}{d^2k\,dy}}{A\,\dfrac{d\sigma^{pp}}{d^2k\,dy}}$$



We can plot it for the quasi-classical cross section calculated before (Y.K., A. M. '98):

$$R^{pA}(k_T) = \frac{k^4}{Q_S^4}\left\{-\frac{1}{k^2} + \frac{2}{k^2}e^{-k^2/Q_S^2} + \frac{1}{Q_S^2}e^{-k^2/Q_S^2}\left[\ln\frac{Q_S^4}{4\Lambda^2 k^2} + Ei\left(\frac{k^2}{Q_S^2}\right)\right]\right\}$$

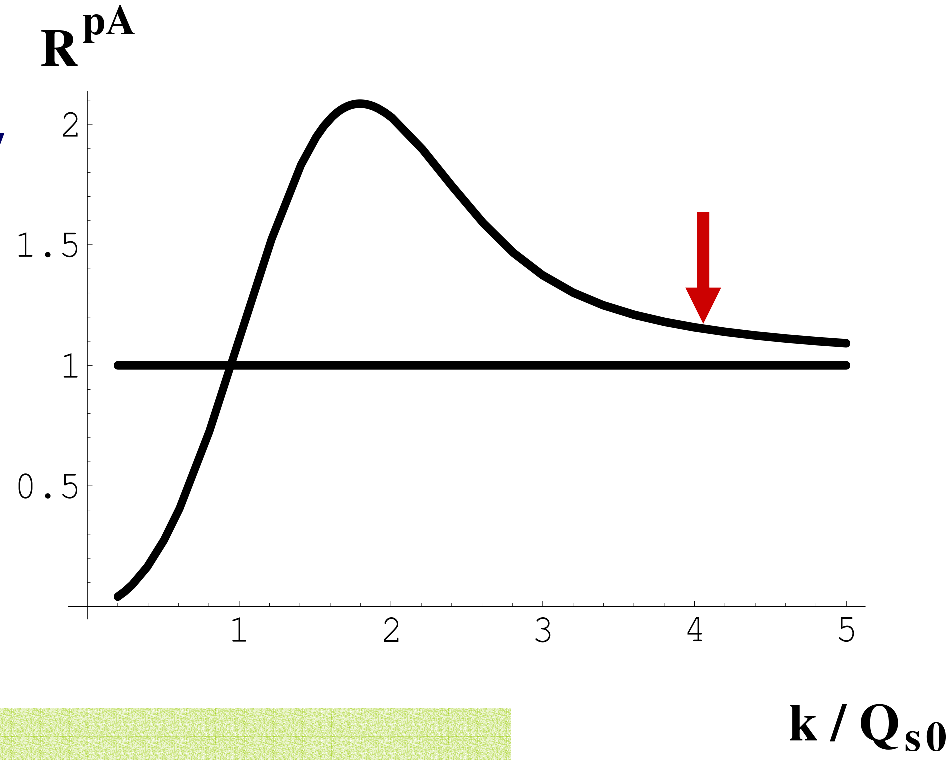
Classical gluon production leads to Cronin effect!
Nucleus pushes gluons to higher transverse momentum!

Kharzeev
Yu. K.
Tuchin '03

(see also Kopeliovich et al, '02; Baier et al, '03; Accardi and Gyulassy, '03)

Analyzing Cronin Effect

- To prove that Cronin effect actually does take place one has to study the behavior of R^{pA} at large k_T (cf. Dumitru, Gelis, Jalilian-Marian, quark production, '02-'03):



Note the sign!

$$R^{pA}(k_T) = 1 + \frac{3}{2}\frac{Q_S^2}{k^2}\ln\frac{k^2}{\Lambda^2} + \ldots, \qquad k_T \to \infty$$

R^{pA} approaches 1 from above at high p_T ⇨ there is an enhancement!

Cronin Effect

$$R^{pA}(k_T) = 1 + \frac{3}{2}\frac{Q_S^2}{k^2}\ln\frac{k^2}{\Lambda^2} + \ldots, \qquad k_T \to \infty$$

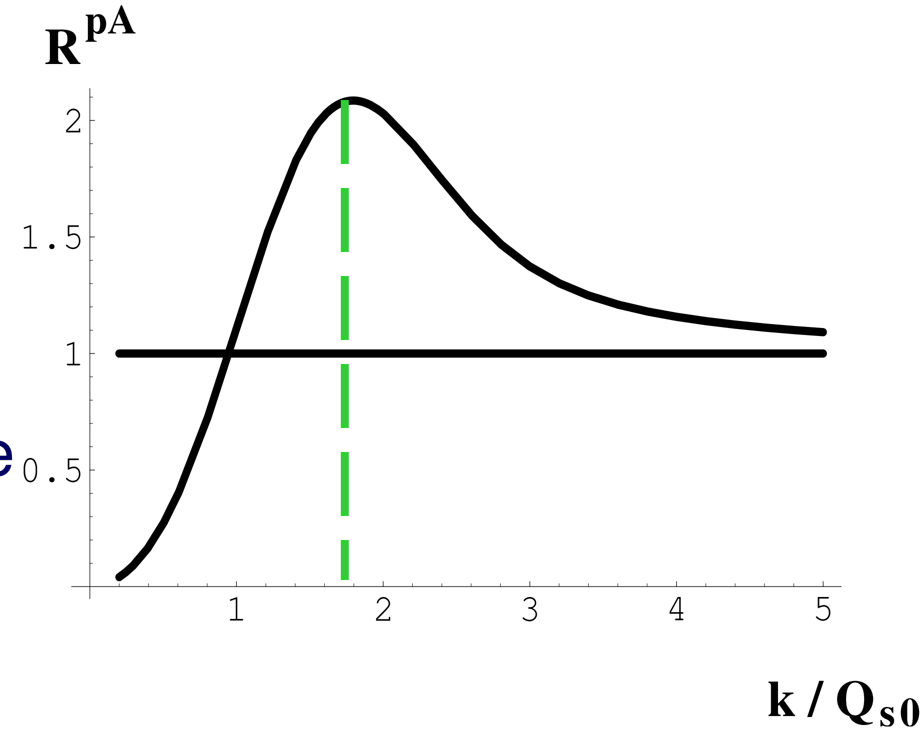
The position of the Cronin maximum is given by

$$k_T \sim Q_S \sim A^{1/6}$$

as $Q_S^2 \sim A^{1/3}$.

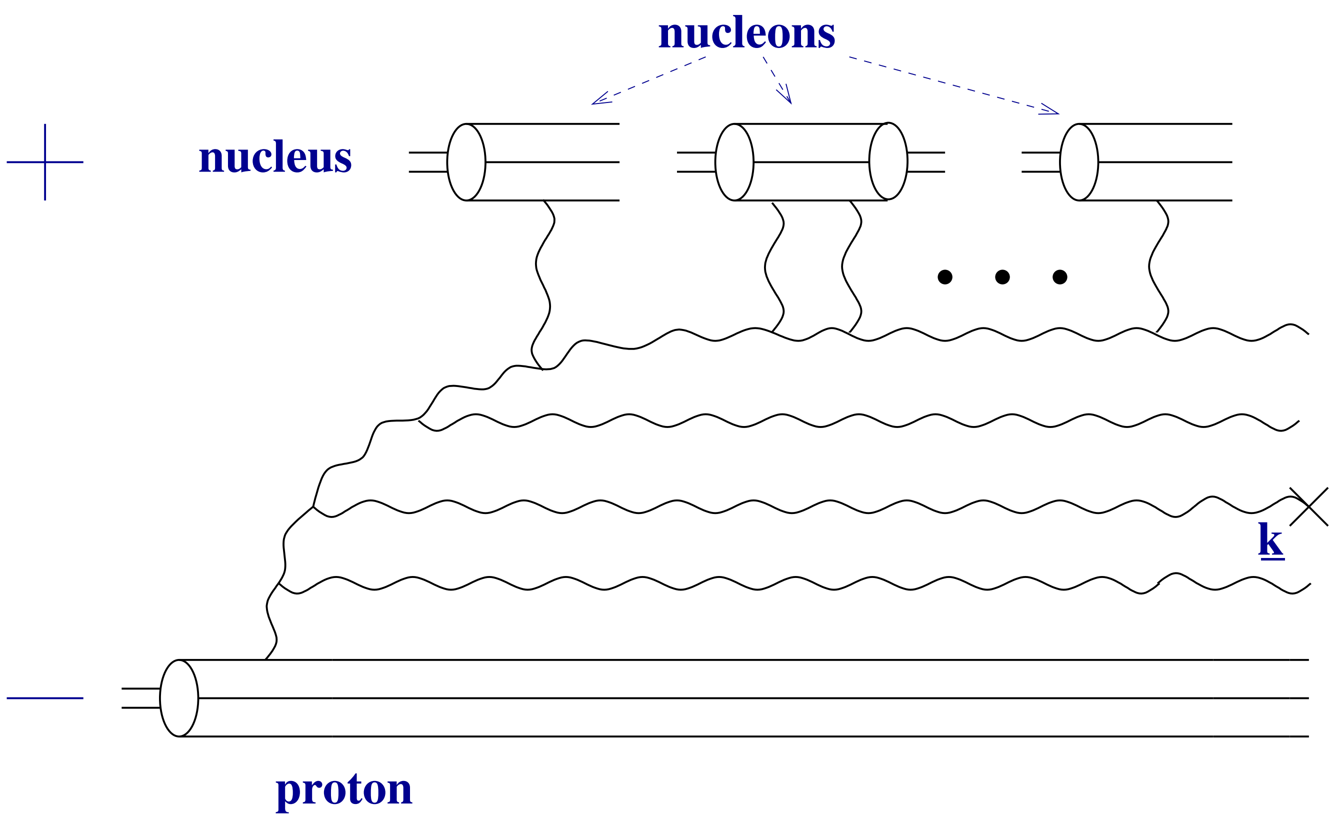
Using the formula above we see that the height of the Cronin peak is

$$R^{pA}(k_T = Q_S) \sim \ln Q_S \sim \ln A.$$



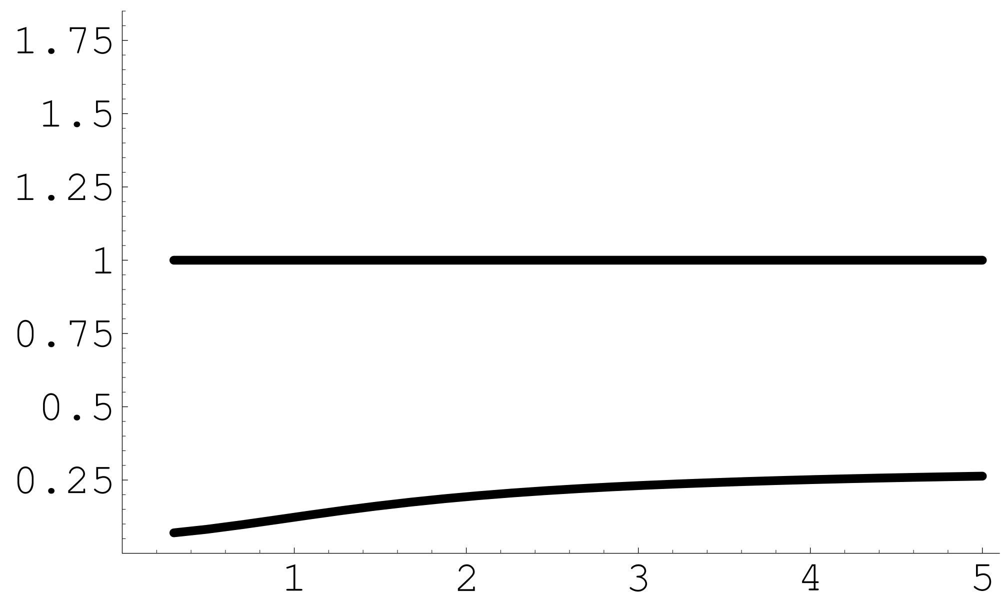
⇨ The height and position of the Cronin maximum are increasing functions of centrality (A)!

Including Quantum Evolution



To understand the energy dependence of particle production in pA one needs to include quantum evolution resumming graphs like this one. It resums powers of $\alpha \ln 1/x = \alpha Y$.

(Yu. K., K. Tuchin, '01)



1.75
1.5
1.25
1
0.75
0.5
0.25
1
2
3
4
5

ERROR: invalidrestore
OFFENDING COMMAND: restore

STACK:

-savelevel-
-savelevel-